Rossmoyne Senior High School

WA Exams Practice Paper A, 2015

Question/Answer Booklet

MATHEMATICS APPLICATIONS UNITS 1 AND 2

Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section Two: Calculator-assumed **(98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(7 marks)**

A machine produces nails with lengths that follow a normal distribution with a mean of 36 mm and a standard deviation of 0.5 mm.

(a) State the probability that a randomly chosen nail has a length

(i) less than 35.3 mm (1 mark)

$P(X < 35.3) = 0.081$

(ii) exactly 35.3 mm (1 mark)

$P(X = 35.5) = 0$

(iii) between 35.3 mm and 36.3 mm (1 mark)

$P(35.5 < X < 36.3) = 0.645$

(b) The longest 5% of nails are discarded. What length do the discarded nails exceed? (1 mark)

$P(X > x) = 0.05 \Rightarrow x = 36.8 \text{ mm}$

(c) 2500 nails are produced, 1800 of which have lengths within x mm of the mean. Determine the value of x. (3 marks)

$$P(36 - x < X < 36 + x) = \frac{1800}{2500}$$
$$= 0.72$$

$$P(X > 36 + x) = \frac{1 - 0.72}{2}$$
$$= 0.14$$

$$36 + x = 36.54$$
$$x = 0.54 \text{ mm}$$

See next page

Question 9 **(6 marks)**

(a) The price of a sought-after classic car was increased from \$16 495 to \$19 995.

(i) By what percentage was the price increased? (2 marks)

$19995 - 16495 = 3500$
$3500 \div 16495 \times 100 \approx 21\%$

(ii) The dealer who sold the car was paid a 2.2% commission on the increased price. How much was this commission? (1 mark)

$19995 \times 2.2 \div 100 = \$439.89$

(b) In a clothing store sale, all prices were reduced by 15%.

(i) Calculate the sale price of a shirt originally costing \$48. (1 mark)

$100 - 15 = 85$
$48 \times 85 \div 100 = \$40.80$

(ii) Calculate the original price of a pair of jeans costing \$46.75 in the sale. (2 marks)

$46.75 \div 0.85 = \$55$

See next page

Question 10 **(8 marks)**

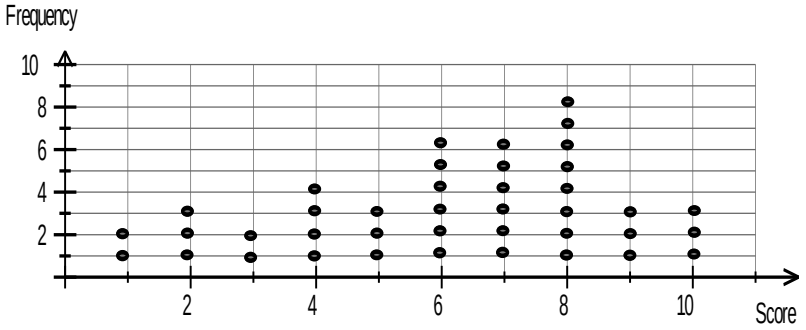
A teacher kept a record of the marks scored by 35 of her students in an arithmetic test. Her record is shown below in the tally table.

Score	Number of pupils	Frequency
1	/ /	**2**
2	/ / /	**3**
3	**/ /**	**2**
4	/ / / /	**4**
5	**/ / /**	**3**
6	~~////~~ **/**	**6**
7	~~////~~ /	**6**
8	~~////~~ / / /	**8**
9	**/ / /**	**3**
10	/ / /	**3**

(a) She still had to add scores of 9, 5, 6, 6 and 3 from five students who had just sat the test.

Add these scores to the tally table above and then complete the frequency column. (2 marks)

(b) On the axes below draw a dot frequency plot to represent the 40 scores. (2 marks)



(c) For all 40 scores, determine

(i) the mean (1 mark)

6.1

(ii) the median (1 mark)

6.5

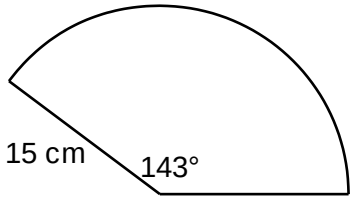
(d) What feature of the dot frequency plot would indicate that the mean would not be the same as the median? Explain your answer. (2 marks)

The shape of the dot frequency plot – because it displays negative skew, then the mean will be less than the median.

See next page

Question 11 **(12 marks)**

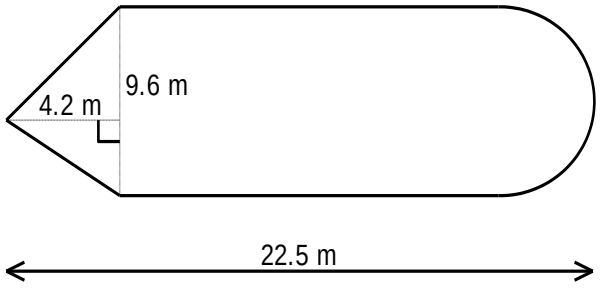
(a) Determine the perimeter of the sector shown below. (2 marks)



$$2 \times \pi \times 15 \times \frac{143}{360} = 37.4$$

$$37.4 + 15 + 15 = 67.4 \text{ cm}$$

(b) A rectangular lawn, with additional triangular and semi-circular ends, has dimensions as shown below. Determine the area of the lawn. (4 marks)



$$\frac{1}{2} \times 9.6 \times 4.2 = 20.16$$

$$r = 9.6 \div 2$$
$$= 4.8$$

$$\frac{1}{2} \times \pi \times 4.8^2 = 36.19$$

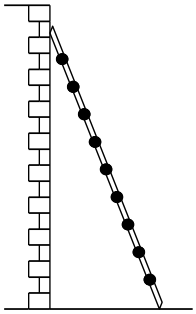
$$9.6 \times (22.5 - 4.2 - 4.8) = 9.6 \times 13.5$$
$$= 129.6$$

$$20.16 + 36.19 + 129.6 = 185.9$$
$$\approx 186 \text{ m}^2$$

See next page

(c) A 2.7 m long ladder leans against a vertical wall as shown. The top of the ladder is 2.4 m above the horizontal ground.

(i) Determine how far the base of the ladder is from the wall, rounded to one decimal place. (2 marks)



$2.7^2 - 2.4^2 = 1.53$

$\sqrt{1.53} = 1.24$ m

(ii) If the top of the ladder slips 30 cm down the wall, calculate how far the base of the ladder slips away from the wall. (4 marks)

$2.4 - 0.3 = 2.1$

$2.7^2 - 2.1^2 = 2.88$

$\sqrt{2.88} = 1.70$

$1.70 - 1.24 = 0.46$ m (or 46 cm)

See next page

Question 12 **(8 marks)**

The table below, based on figures from the Australian Taxation Office, shows the tax rates for the financial year 2014 - 2015.

Annual taxable income	***Annual tax on this income***
\$1 - \$18 200	Nil
\$18 201 - \$37 000	19c for each \$1 over \$18 200
\$37 001 - \$80 000	\$3 572 plus 32.5c for each \$1 over \$37 000
\$80 001 - \$180 000	\$17 547 plus 37c for each \$1 over \$80 000
\$180 001 and over	\$54 547 plus 47c for each \$1 over \$180 000

A single person, who works away from his home on a remote mine site, expects to earn a taxable income of \$11 500 per month throughout this financial year.

(a) Calculate the annual taxable income for this person. (1 mark)

$11500 \times 12 = \$138\ 000$

(b) Calculate the annual tax payable by this person. (3 marks)

$138000 - 80000 = 58000$
$58000 \times 0.37 = 21460$
$17547 + 21460 = \$39\ 007$

The Medicare Levy Surcharge is added to a person's tax if they do not have private health insurance. The surcharge is calculated as a percentage of the annual taxable income using the rates shown in this table:

Taxable income (\$)	1-90 000	90 001-105 000	105 001-140 000	140 001 and over
Surcharge	0%	1%	1.25%	1.5%

(c) This person did not have private health insurance.

(i) Calculate their Medicare Levy Surcharge. (2 marks)

$138000 \times 1.25 \div 100 = \$1\ 725$

(ii) Private health insurance would cost this person \$28.57 per week. Comment on whether this person should consider such insurance from a financial point of view. (2 marks)

They should take out insurance, as the cost of health insurance is \$1485.64, which is less than the surcharge.

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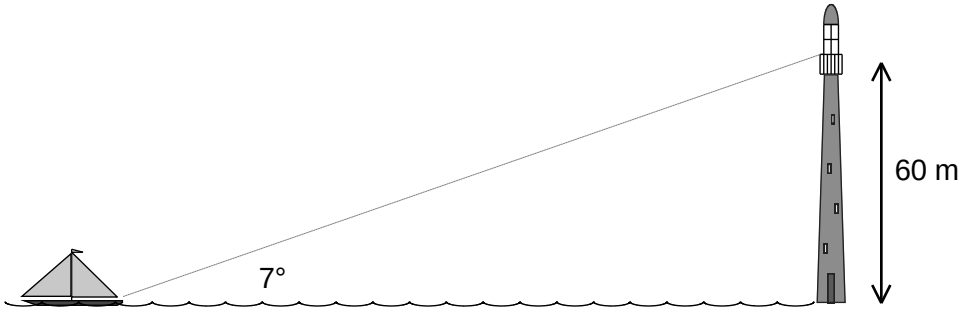
Question 13 **(7 marks)**

A lighthouse keeper looks out to sea and is surprised to see a boat heading directly towards her. To alert the ship she shines a spotlight directly towards the boat from a point 60 m above sea level, as shown below.



(a) If the angle of elevation of the spotlight, as seen from the boat, is 7°, determine the distance from the ship to the base of the lighthouse, rounded to the nearest metre.

(3 marks)

$$\tan 7° = \frac{60}{d}$$

$$d = \frac{60}{\tan 7°}$$

$$= 488.66$$

$$\approx 489 \text{ m, to nearest m.}$$

One minute later, the spotlight is noticed on the boat and it instantly stops.

(b) If the boat was moving with a speed of 3 metres per second, explain why it will it travel 180 m in one minute. (1 mark)

60 seconds in one minute, so
$3 \times 60 = 180$ metres

(c) At the instant the boat stops, determine

(i) the angle of depression of the boat viewed from the top of the lighthouse.

(2 marks)

$$489 - 180 = 309$$

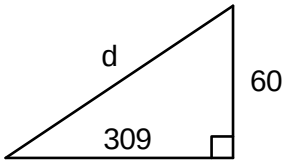
$$\theta = \tan^{-1}\frac{60}{309} = 11°$$

(ii) the direct distance from the boat to the spotlight at the top of the lighthouse.

(1 mark)

$$d = \sqrt{309^2 + 60^2}$$

$$= 315 \text{ m}$$



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Question 14 **(7 marks)**

A snapshot of information about four companies listed on the Australian share market is given in the table below.

Company	ASX Code	Market value of share ($)	Earnings per share (cents)	Price-to-earnings ratio	Annual dividend (cents)	Percentage dividend (%)
Amcor	AMC	11.34	46.8	**A**	43	3.79
Bradken	BKN	4.23	12.7	33.3	26.0	6.15
CSL	CSL	73.50	**B**	25.6	123.73	1.68
Dulux	DLX	5.56	28.2	19.7	**C**	3.51

(a) Determine the values of A, B, and C in the table above. (3 marks)

$1134 \div 46.8 = A \Rightarrow A = 24.2$

$7350 \div B = 25.6 \Rightarrow B = 287.1$

$5.56 \times 3.51\% = C \Rightarrow C = 19.5$

(b) A portfolio of shares with a market value of $12 200 consisted of 200 AMC shares, 200 DLX shares and the remainder being CSL shares. Determine the number of CSL shares in the portfolio. (2 marks)

$200 \times 11.34 + 200 \times 5.56 = 2268 + 1112 = 3380$

$12200 - 3380 = 8820$

$8820 \div 73.5 = 120$ CSL shares

(c) Another portfolio had 500 AMC shares and 200 CSL shares. Determine the total dividend paid on this portfolio. (2 marks)

$500 \times 0.43 = 215$

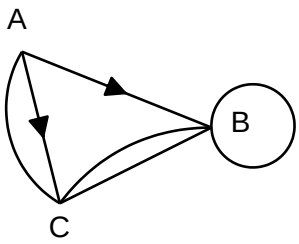
$200 \times 1.2373 = 247.46$

Total $= \$462.46$

See next page

Question 15 **(6 marks)**

A system of roads connecting the locations A, B and C is shown below. The roads from A to C and from A to B can only be travelled in the direction indicated, but all other roads are two way.



(a) Complete the table below to show the number of one stage routes from location to location. (2 marks)

		To		
		A	B	C
From	A	0	1	2
	B	**0**	**2**	**2**
	C	**1**	**2**	**0**

(b) Arrange the information from the table in (a) as a matrix M. (1 mark)

$$M = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 2 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

(c) Determine M^2. (1 mark)

$$M^2 = \begin{bmatrix} 2 & 6 & 2 \\ 2 & 8 & 4 \\ 0 & 5 & 6 \end{bmatrix}$$

(d) How many two stage routes are there from C to B? (1 mark)

5

(e) How many three stage routes are there from A to B? (1 mark)

$$M^3 = \begin{bmatrix} 2 & 18 & 16 \\ 4 & 26 & 20 \\ 6 & 22 & 10 \end{bmatrix}$$

So 18 three stage routes.

See next page

Question 16 **(7 marks)**

(a) A woman qualifies for an age pension of $536.70 per fortnight, so long as she does not earn more than $268 in that time. In any fortnight that she does earns more than $268, her pension will be reduced by 50 cents in the dollar for earnings over $268.

The woman starts a part time job for 12 hours per week that pays $17.89 per hour.

(i) Calculate her fortnightly pension. (3 marks)

$12 \times 17.89 \times 2 = 429.36$

$429.36 - 268 = 161.36$

$161.36 \times 0.50 = 80.68$

$536.70 - 80.68 = \$456.02$

(ii) How much is her fortnightly income? (1 mark)

$429.36 + 456.02 = \$885.38$

(b) An online store in the US sells a 185g tube of skin cream for US$24.99.

(i) How much does the tube cost in Australian dollars, if the exchange rate for one A$ is 84 US cents? (1 mark)

$24.99 \div 0.84 = A\$29.75$

(ii) The store also sells a 330g tube of the same cream for U$44.99. Determine which size of tube represents the best value. (2 marks)

$185 \div 24.99 = 7.4$ g/$

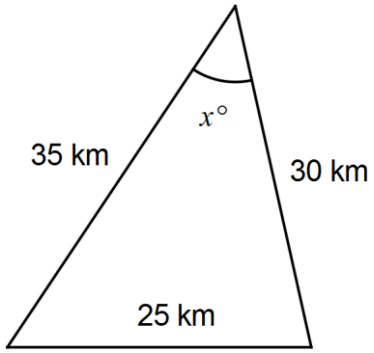
$330 \div 44.99 = 7.3$ g/$

Smaller tube represents best value.

See next page

Question 17 **(7 marks)**

An aeroplane flew 25 km due west from an airfield. It then turned and flew a further 35 km before making a final turn and returning to the airfield, as shown in the sketch below.



(a) Using trigonometry, determine the size of the angle marked x in the sketch. (2 marks)

$$\cos x° = \frac{35^2 + 30^2 - 25^2}{2 \times 35 \times 30}$$
$$x° = 44.4°$$

(b) Using trigonometry, determine the size of the angle opposite the 30 km side to the nearest degree. (2 marks)

$$\frac{25}{\sin 44.4} = \frac{30}{\sin \theta}$$
$$\theta = 57.1°$$
$$\approx 57°$$

(c) Determine the direction that the plane flew

(i) along the 35 km leg of the flight as a three-figure bearing. (1 mark)

$$90 - 57 = 33 \Rightarrow 033°$$

(ii) along the 30 km leg of the flight as a compass bearing. (2 marks)

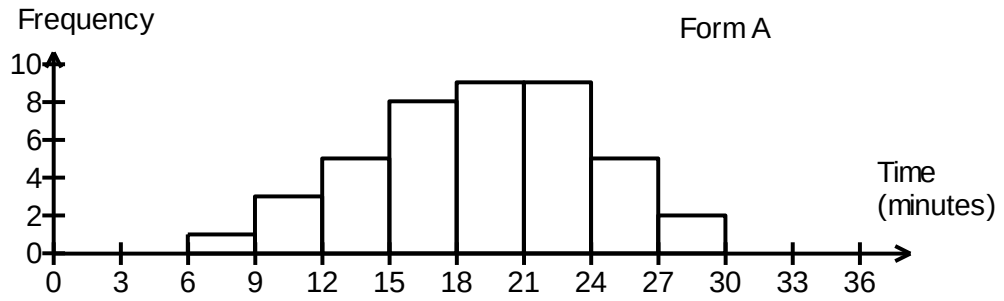
$$44.4 - 33 = 11.4 \Rightarrow \text{S}11.4°\text{E}$$

See next page

Question 18 **(9 marks)**

A local council is interested in what the local ratepayers think of the services it provides. The council is considering two different feedback forms, Form A and Form B. It has decided to trial each form using a random sample of 50 ratepayers drawn from the 6475 in its database.

The council sent out copies of Form A to a sample of 50 ratepayers and 42 of these were returned by the required date. Each ratepayer was asked how long they took to complete the form and the times are summarised in the histogram below.

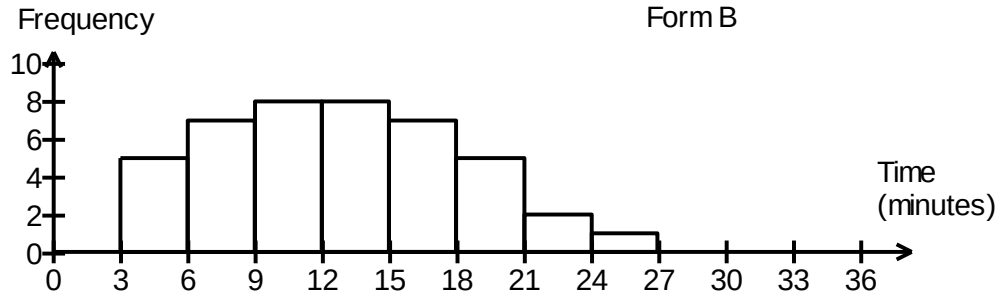


(a) Calculate the mean and standard deviation for the time taken to complete Form A. (2 marks)

Mean = 19.1 minutes

SD = 5.0 minutes

The council also sent out copies of Form B to another sample of 50 ratepayers and 43 of these were returned by the required date. The mean time to complete Form B was 12.8 minutes, with a standard deviation of 5.41 minutes. The times are summarised in the histogram below.



(b) Compare the modality and shape of the distributions shown in the histograms for Form A and Form B. (2 marks)

Both distributions are unimodal.

The data for Form A displays negative skew whilst that for Form B displays positive skew.

See next page

(c) Explain which of the two forms takes the longest to complete. (1 mark)

Form A takes longest, as it has a greater mean time (19.1 minutes) compared to Form B (12.8 minutes).

(d) Which form has more variation in completion time? Justify your answer with reference to appropriate statistics. (2 marks)

Form B has slightly more variation in times with a standard deviation of 5.41 minutes compared to only 5.0 minutes for Form A.

(e) The council know from past experience that the quicker a form is to complete, the more responses they are likely to receive and that the less variation in time to complete a form, the more reliable the responses are. Which form would you recommend the council use? Justify your answer. (2 marks)

Form B has a mean time that is considerably less than that of Form A (6.3 minutes less) which means the council are more likely to receive more Form B's than A's.

Form A has a slightly smaller sd than Form B (0.4 minutes less) which means A will generate slightly more reliable responses.

The small difference in sd's could be a result of the sampling and so recommend that the council use Form B as it is more likely to be returned and its reliability is similar to that of Form A.

See next page

Question 19 **(7 marks)**

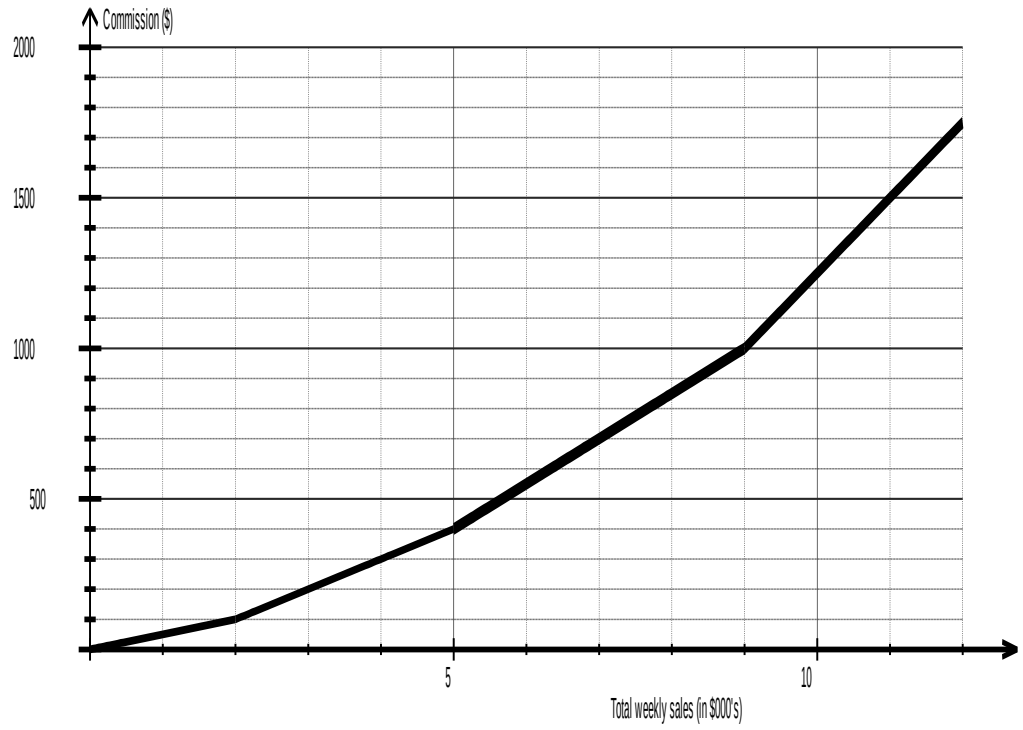
Employees of a sales company are paid commissions based on their total weekly sales, as shown in the table below.

Total weekly sales ($)	Commission
0 - 2 000	5c for each $1
2 001 - 5 000	$100 plus 10c for each $1 over $2 000
5 001 - 9 000	$400 plus 15c for each $1 over $5 000
9 000 and over	$1 000 plus 25c for each $1 over $9 000

(a) Determine the commission paid to an employee who has total weekly sales of $6 500. (2 marks)

$$400 + 0.15(6500 - 5000) = \$625$$

(b) Complete the piecewise graph below to show the commission on total weekly earnings from $0 to $12 000. (2 marks)



(c) At the end of one week, an employee was paid a commission of $285.

(i) Use the graph to estimate the total weekly sales for this employee to the nearest thousand dollars. (1 mark)

$4 000

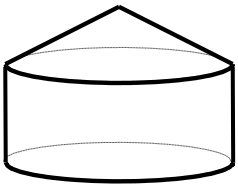
(ii) Use information from the table to calculate the total weekly sales for this employee. (2 marks)

$$100 + 0.10(x - 2000) = 285$$
$$x = \$3850$$

See next page

Question 20 **(7 marks)**

The solid figure below consists of a cylinder of radius 12cm and height 7cm with a cone of the same radius and slant height 12.5cm placed on top.



(a) Determine the total surface area of this solid figure. (3 marks)

Cylinder (curved side and circular base)
$2\pi \times 12 \times 7 + \pi \times 12^2 = 980.18 \text{ cm}^2$

Cone (curved area only)
$\pi \times 12 \times 12.5 = 471.24 \text{ cm}^2$

TSA
$980.18 + 471.24 = 1451.42 \approx 1451 \text{ cm}^2$

(b) Show that the total height of the solid figure is exactly 10.5cm. (2 marks)

Using $r^2 + h^2 = s^2$ for cone:
$12^2 + h^2 = 12.5^2 \Rightarrow h = 3.5 \text{ cm}$

Total height is $7 + 3.5 = 10.5 \text{ cm}$.

(c) Determine the total volume of this solid figure. (2 marks)

Cylinder
$\pi \times 12^2 \times 7 = 3166.73 \text{ cm}^3$

Cone
$\frac{1}{3} \times \pi \times 12^2 \times 3.5 = 527.79 \text{ cm}^3$

Total volume
$3166.73 + 527.79 = 3694.52 \approx 3695 \text{ cm}^3$

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

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